

Particle-Number-Conditioned Master Equation and Its Application in Quantum Measurement and in Quantum Transport

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Outline:

- ◆ **Master equation approach to quantum transport**
- ◆ **Quantum measurement of solid-state qubit**
 - **Example: SET detector, signal-to-noise ratio, etc**
- ◆ **Quantum transport**
 - **Current fluctuation, full counting statistics**
 - **Example: double-dot interferometer**

量子输运主方程方法

量子输运中的主方程方法 (一些背景说明)

经典rate方程: 1990'年代, 共振隧穿系统, 库伦阻塞系统
(电流台阶, 电流噪声)

优点: 相对 nGF方法和 Landauer-Buttiker理论,
简洁直观, 物理意义非常清楚

缺点: 量子相干性不能(不易)纳入其中

量子rate方程: Gurvitz (1996年), 直接从多粒子(即包含“运输系统+电极”) Schrodinger方程出发, 导出一个可以包含多体相互作用和量子相干性的新方法

基本特征: “隧穿电子数”相关的“运输系统约化状态”的运动方程

缺点: 零温度, 大电压极限

现状: 近年来变得比较流行了, 例如: 1) 丹麦Jauho小组用该方法系统研究了quantum shuttle 运输问题; 2) 欧洲其他几个小组用该方法研究了量子点输运中的全计数统计问题

条件量子（运输）主方程



量子耗散

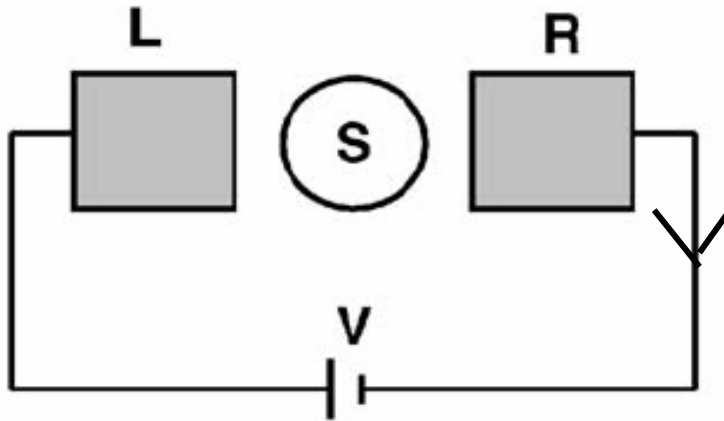
$$\dot{\rho} = -i[H_S, \rho] - R\rho$$

X.Q. Li *et al*:

PRL 94, 066803 (2005)

PRB 71, 205304 (2005)

PRB 76, 085325 (2007)



量子输运

$$\dot{\rho} = -i[H_S, \rho] - R\rho$$

$$\dot{\rho}^{(n)} = -i[H_S, \rho^{(n)}] - [R_1\rho^{(n)} + R_2\rho^{(n+1)} + R_3\rho^{(n-1)}]$$

Current

$$\begin{aligned} I(t) &\equiv e \frac{dN(t)}{dt} = e \sum_n n \text{Tr}[\dot{\rho}^{(n)}(t)] \\ &= \frac{e}{2} \text{Tr}[\bar{Q}\rho Q + \text{H.c.}], \end{aligned}$$

where $\bar{Q} \equiv \tilde{Q}^{(-)} - \tilde{Q}^{(+)}$

Noise spectrum: MacDonald's formula

$$S_I(\omega) = 2\omega \int_0^\infty dt \sin \omega t \frac{d}{dt} [e^2 \langle n^2(t) \rangle - (\bar{I}t)^2]$$

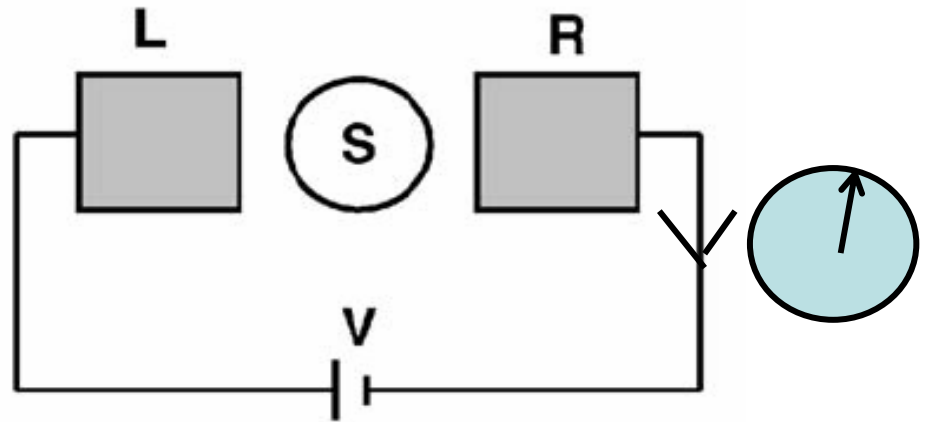
$$\langle n^2(t) \rangle = \sum_n n^2 P(n, t)$$

$$\frac{d}{dt} \langle n^2(t) \rangle = \text{Tr} \left[\bar{Q} \hat{N}(t) Q + \frac{1}{2} \tilde{Q} \rho(t) Q + \text{H.c.} \right]$$

$$\hat{N}(t) \equiv \sum_n n \rho^{(n)}(t) \quad \frac{d\hat{N}}{dt} = -i\mathcal{L}\hat{N} - \frac{1}{2}\mathcal{R}\hat{N} + \frac{1}{2}(\bar{Q}\rho Q + \text{H.c.})$$

Full counting statistics

Levitov, Lee, & Lesovik:
J. Math. Phys. 37, 4845(1996)



Generating function:
$$e^{-F(\chi)} = \sum_n P(n, t) e^{in\chi}$$

$$C_k = -(-i\partial_\chi)^k F(\chi) |_{\chi=0}$$

$$C_1 = \bar{n}$$

$$I = eC_1/t$$

Current

$$C_2 = \overline{n^2} - \bar{n}^2$$

$$S = 2e^2 C_2/t$$

Zero-frequency noise

$$C_3 = \overline{(n - \bar{n})^3}$$

$$F = C_2/C_1$$

Fano factor

$$\dot{\rho}^{(n)} = A\rho^{(n)} + C\rho^{(n+1)} + D\rho^{(n-1)}$$

$$S(\chi, t) = \sum_n \rho^{(n)}(t) e^{in\chi}$$

$$e^{-F(\chi)} = \text{Tr}[S(\chi, t)]$$

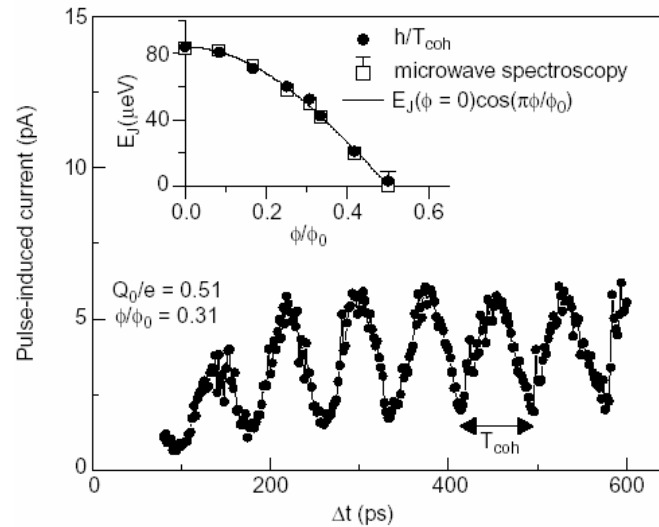
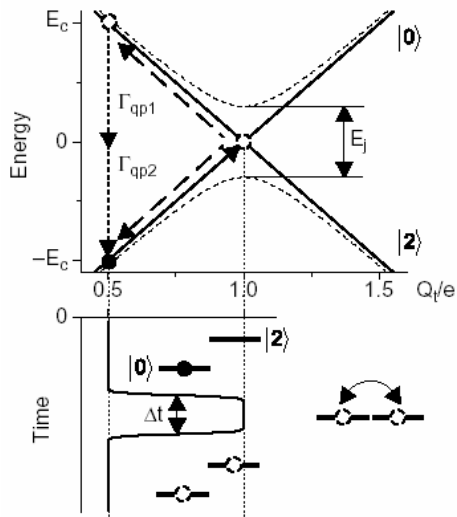
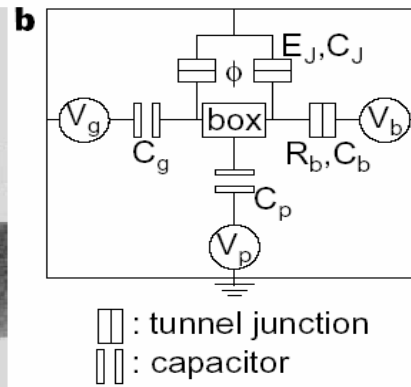
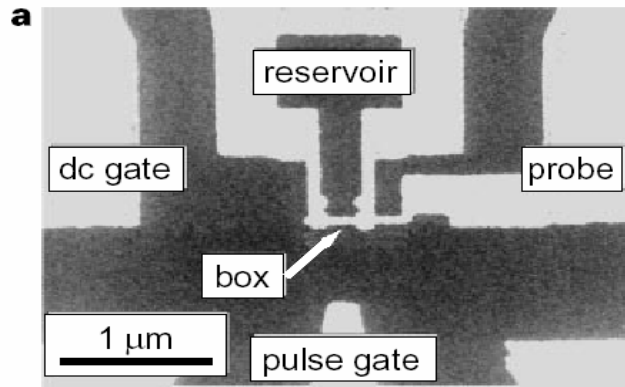
$$\dot{S} = AS + e^{-i\chi}CS + e^{i\chi}DS \equiv \mathcal{L}_\chi S$$

$$F(\chi) = -\lambda_1(\chi)t \quad \lambda_1(\chi)|_{\chi \rightarrow 0} \rightarrow 0$$

量子测量方面的应用

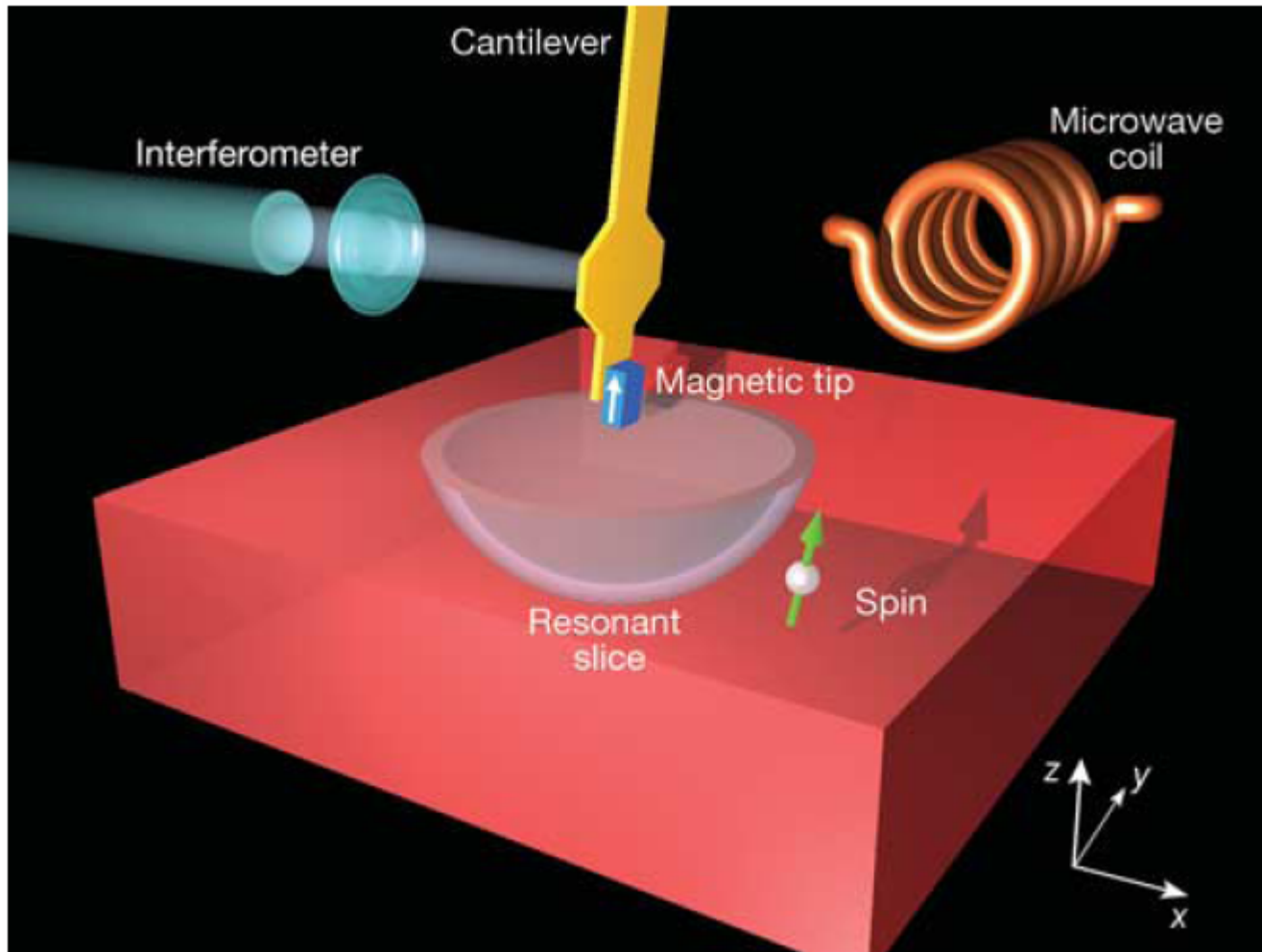
超导量子比特的量子测量（相干振荡）

Nakamura et al, Nature 398, 786 (1999)



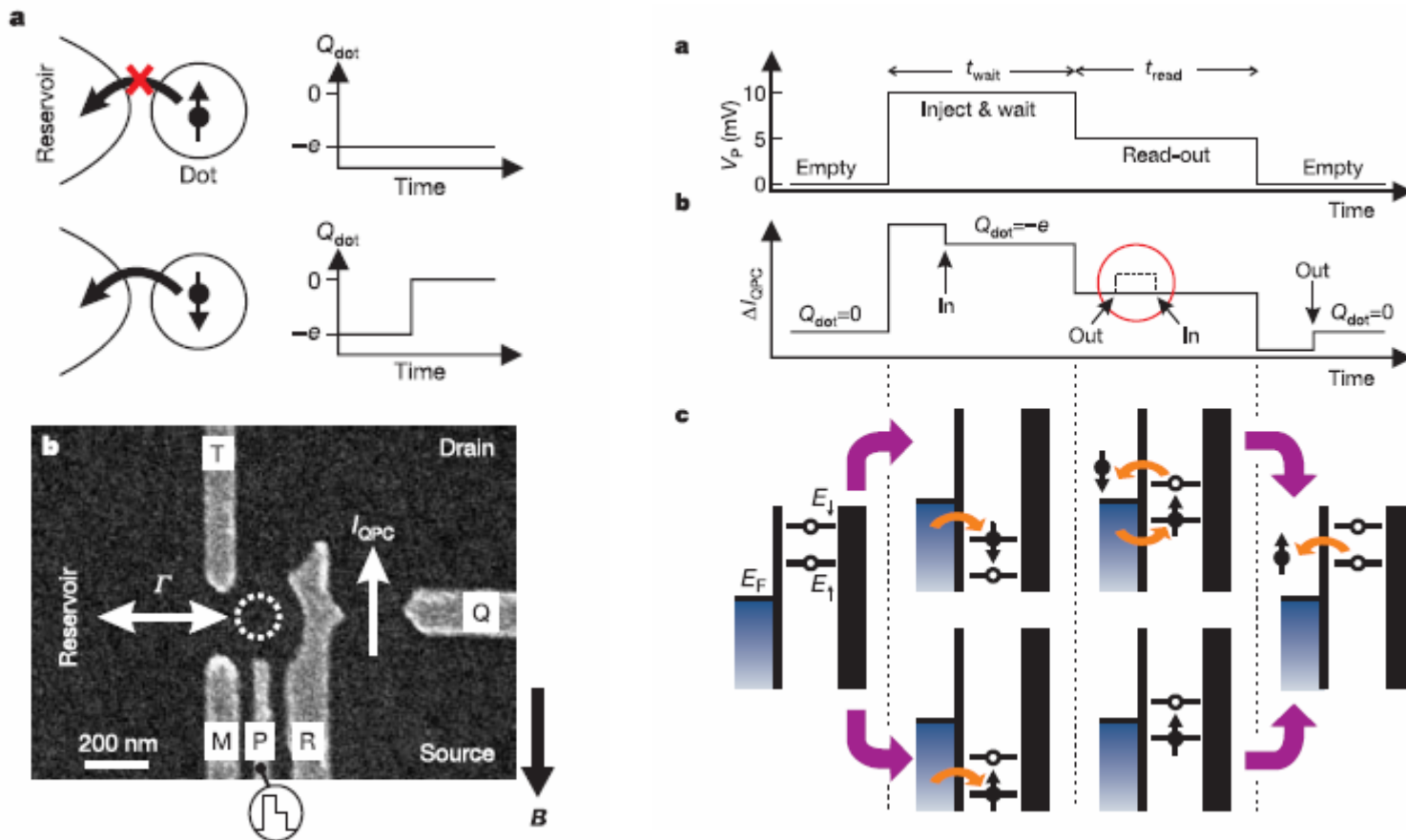
(磁共振力显微镜) 对 (单) 电子自旋的测量

Nature 430, 329 (2004)



QPC 对单电子态的灵敏测量

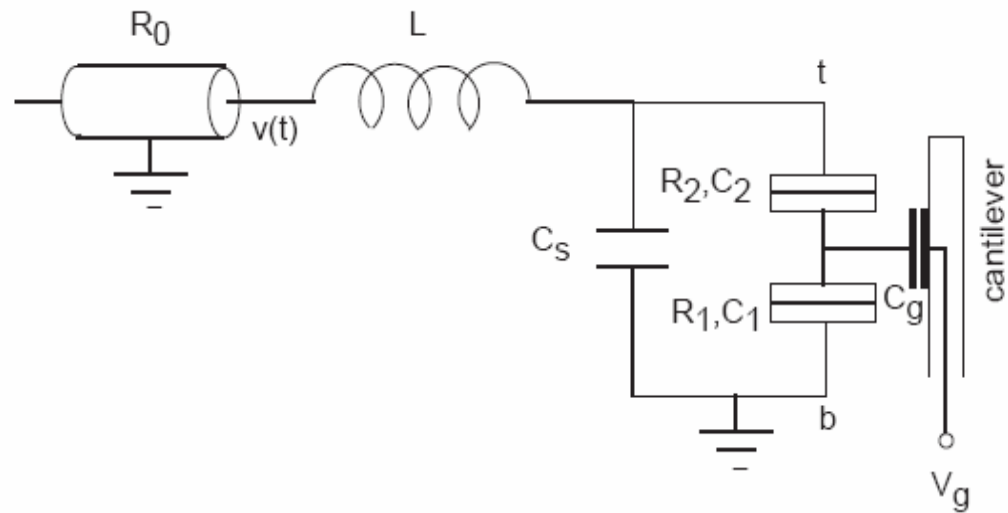
L.P. Kouwenhoven, et al., Nature 430, 431 (2004)



本实验演示了QPC运输电流对单个电子占据的分辨能力，并可从单个电荷占据情况推测单电子自旋态。

(射频) SET的量子测量

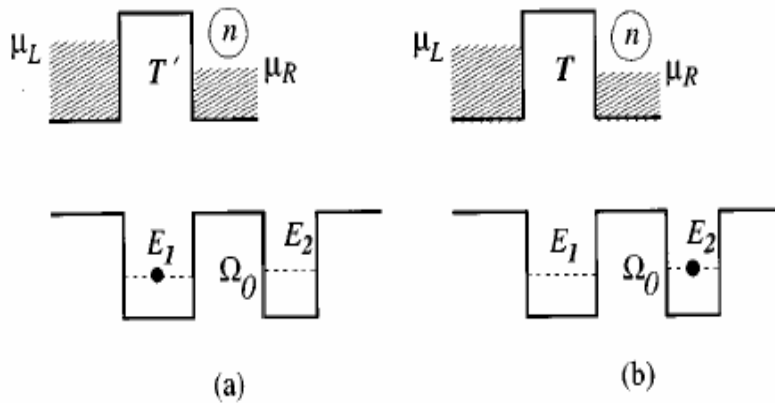
M. Blencowe / Physics Reports 395 (2004) 159–222



Circuit diagram of the rf-SET displacement detector.

QPC/SET测量固态（电荷）量子比特

Gurvitz: PRB 56, 15215 (1997)



Non-trivial points:

- signal-to-noise ratio
- quantum efficiency of meas.
- quantum trajectory under meas.

Schoen: PRB (98); RMP(00); PRL(02)

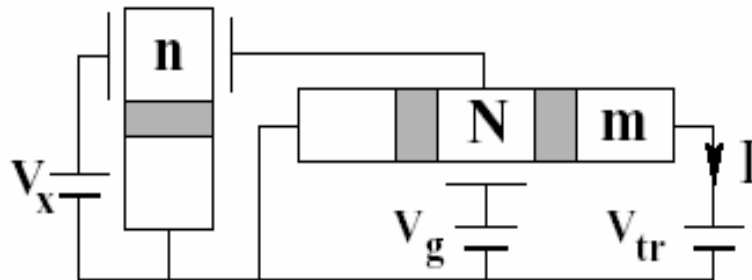


FIG. 1. The circuit of a qubit and a SET used as a meter.

几个典型研究组的工作

1. Gurvitz *et al*: “n”-resolved Bloch Equation

PRB 56, 15215 (1997); PRL85, 812 (2000)
PRL89,018301(2002); PRL91,066801 (2003)

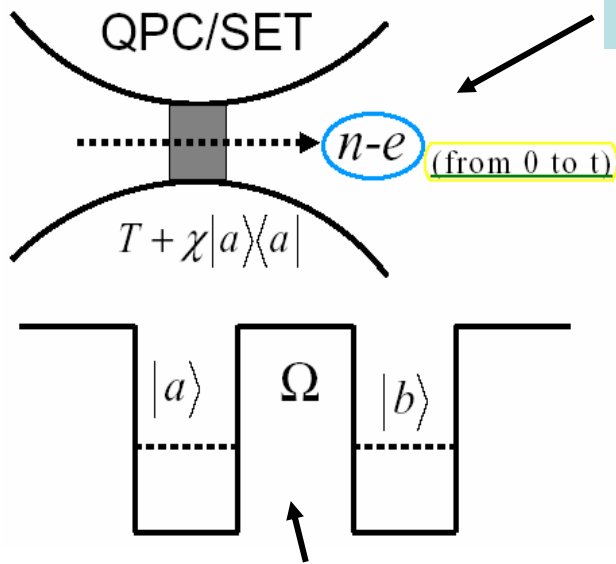
2. Goan, Wiseman and Milburn: quantum trajectory

PRB 63, 125326 (2001); 64, 235307 (2001).

3. Korotkov, Averin *et al*: Bayesian Approach

PRB 60, 5737 (1999) PRB 63, 085312 (2001)
PRB 64, 165310 (2001) PRB 67, 075303 (2003)

Remarks: 以上几个理论方案实际上是严格等价的！

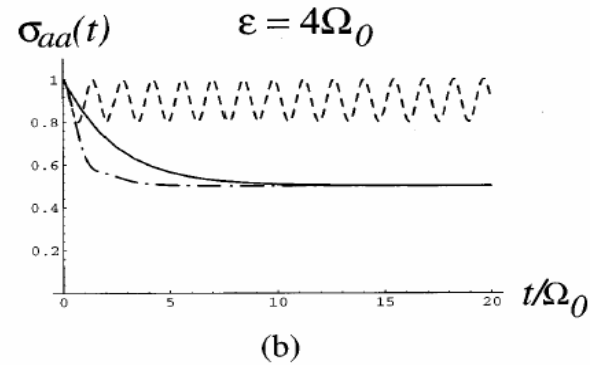
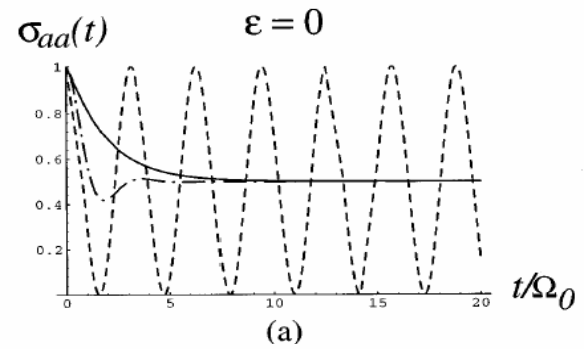
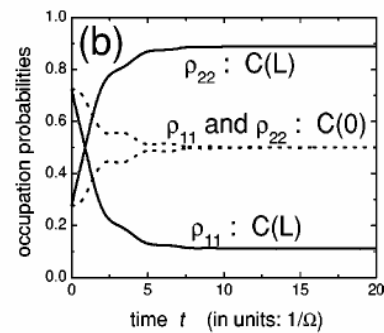
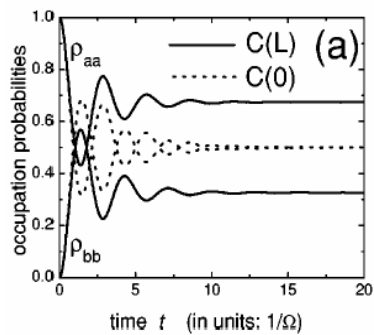
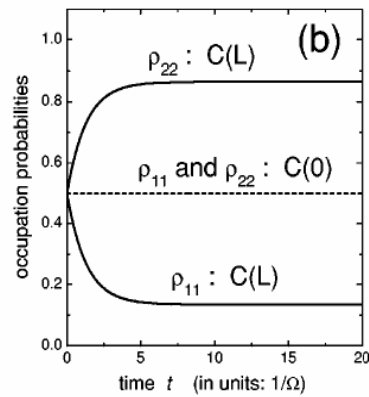
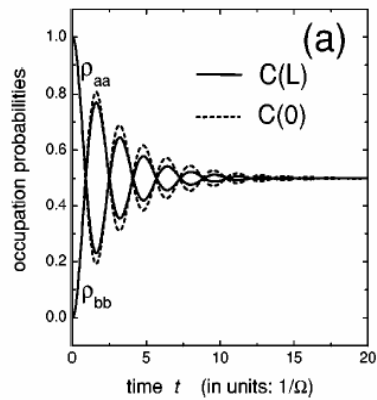


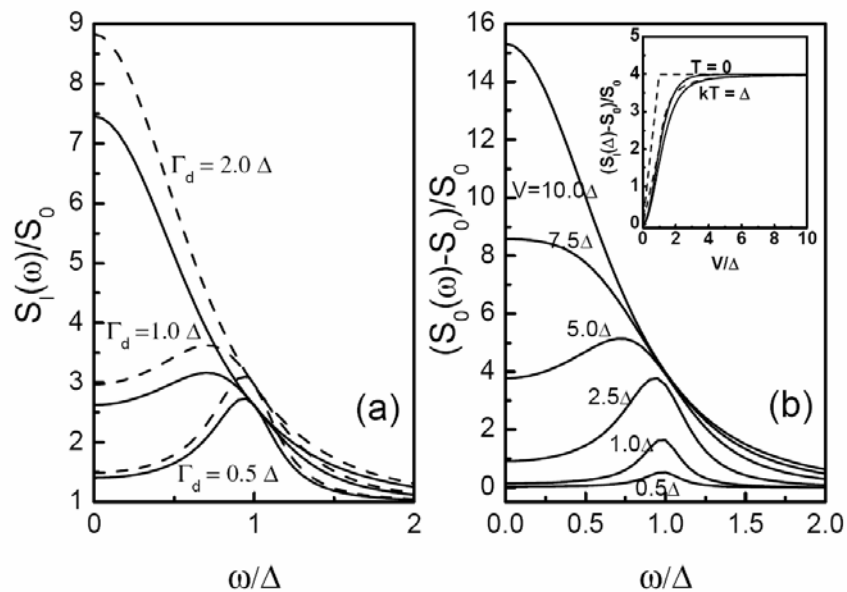
测量仪器 (量子点接触)

**X.Q.Li *et al*, PRB69, 085315 (2004);
PRL94, 066803 (2005).**

$$\dot{\rho}^{(n)} = -i\mathcal{L}\rho^{(n)} - \frac{1}{2} \left\{ [Q\tilde{Q}\rho^{(n)} + \text{H.c.}] - [\tilde{Q}^{(-)}\rho^{(n-1)}Q + \text{H.c.}] - [\tilde{Q}^{(+)}\rho^{(n+1)}Q + \text{H.c.}] \right\}$$

固态量子比特 (耦合量子点模型)

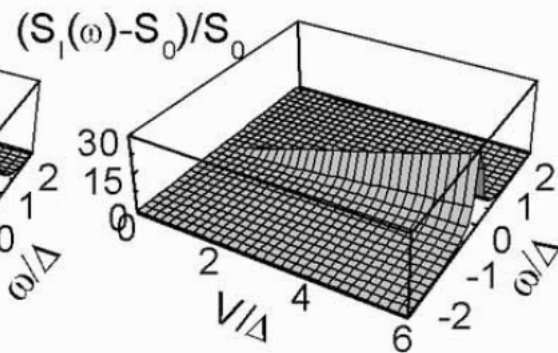
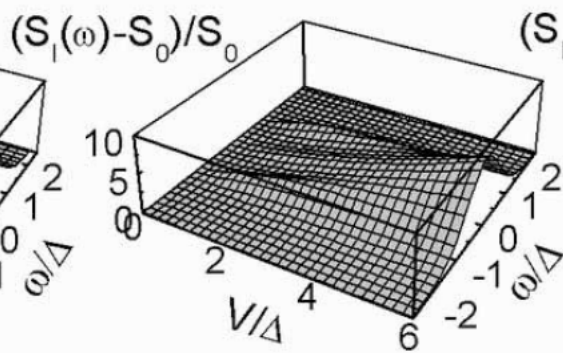
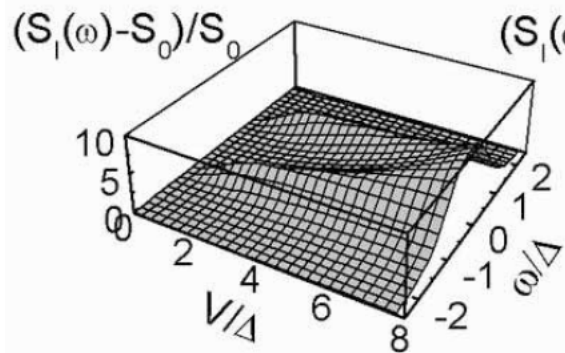




(a) $\theta = \pi/2$

(b) $\theta = \pi/3$

(c) $\theta = \pi/6$



其他相关工作：任意电压下的量子测量

T. M. Stace and S. D. Barrett,
Phys. Rev. Lett. 92, 136802 (2004)

D.V. Averin and A. N. Korotkov,
Phys. Rev. Lett. 94, 069701 (2005);
T. M. Stace and S. D. Barrett,
Phys. Rev.Lett. 94, 069702 (2005).

Application to qubit measurements:

Xin-Qi Li *et al* : **Quantum measurement of a solid-state qubit: A unified quantum master equation approach**, Phys. Rev. B 69, 085315 (2004)

Xin-Qi Li *et al* : **Spontaneous Relaxation of a Charge Qubit under Electrical Measurement**, Phys. Rev. Lett. 94, 066803 (2005).

X.N. Hu *et al*: **Quantum measurement of an electron in disordered potential**, Phys. Rev. B 73, 035320 (2006).

J.S. Jin *et al*: **Quantum coherence control of solid-state charge qubit by means of a sub-optimal feedback algorithm**, PRB 73, 233302 (2006).

S.K. Wang *et al*: **Continuous weak measurement and feedback control of a solid-state charge qubit: physical unravelling of non-Lindblad master equation**, Phys. Rev. B 75, 155304 (2007).

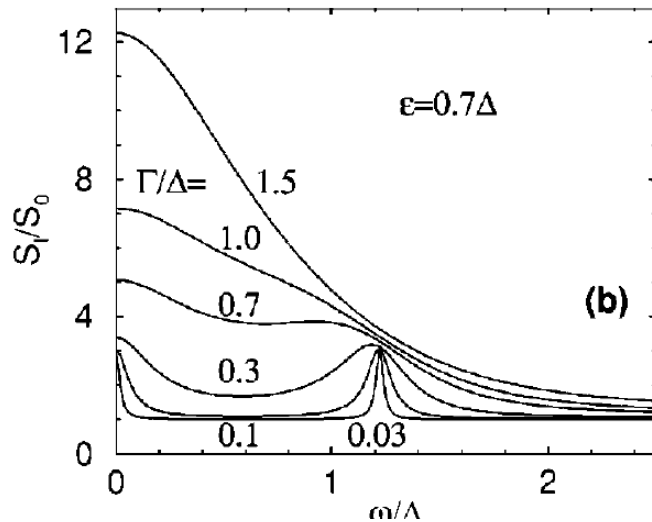
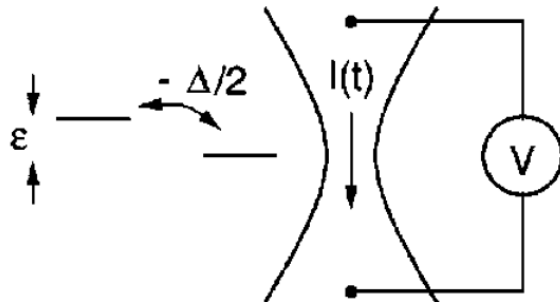
H.J. Jiao *et al*: **Quantum measurement characteristics of double-dot single electron transistor**, Phys. Rev. B 75, 155333 (2007).

Continuous weak measurement of quantum coherent oscillations

A. N. Korotkov and D. V. Averin

Department of Physics and Astronomy, SUNY Stony Brook, Stony Brook, New York 11794-3800

(Received 22 February 2001; revised manuscript received 8 May 2001; published 5 October 2001)

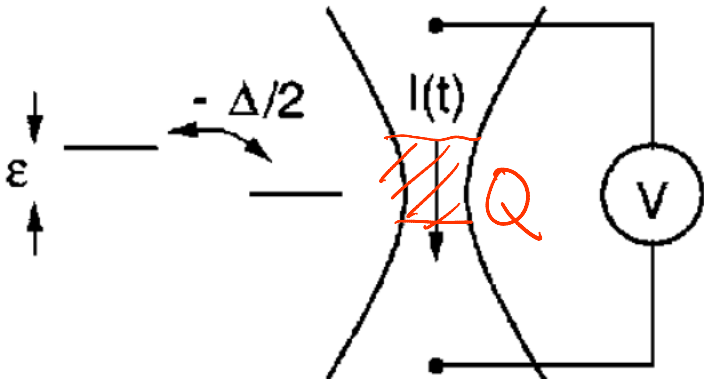


K-A bound:

1) It is shown that **the interplay between the information acquisition and the backaction dephasing of the oscillations by the detector imposes a fundamental limit, equal to “4”, on the signal-to-noise ratio of the measurement.**

2) **The limit is universal, e.g., independent of the coupling strength between the detector and system, and results from the tendency of quantum measurement to localize the system in one of the measured eigenstates.**

A. N. Jordan and M. Buttiker, Phys. Rev. Lett. 95, 220401 (2005).



$$H_{\text{int}} = \sigma_z Q$$

$$Q = \sum_{ij} Q_{ij} \sum_{kp} a_{ik}^+ a_{jp}$$

$$\Gamma = S_Q / (2\hbar^2)$$

Linear Response Theory:

$$O = I + \lambda \sigma_z / 2$$

$$\lambda = -2 \text{Im} S_{QI} / \hbar$$

$$S(\omega) = S_I + (\lambda^2 / 4) S_{zz}(\omega)$$

$$S_{zz}(\tau) = \langle \sigma_z(\tau) \sigma_z \rangle$$

$$S(\omega) = S_I + \frac{\lambda^2 \Gamma}{2} \frac{\Omega^2}{(\omega^2 - \Omega^2)^2 + \omega^2 \Gamma^2}$$

$$S_{\max} = \lambda^2 / (2\Gamma) = \hbar^2 \lambda^2 / S_Q$$

$$\lambda = -2 \operatorname{Im} S_{QI} / \hbar$$

$$\hbar^2 \lambda^2 = 4(\operatorname{Im} S_{QI})^2 \leq 4|S_{QI}|^2 \leq 4S_Q S_I$$

$$\mathcal{R} = S_{\max} / S_I \leq 4$$

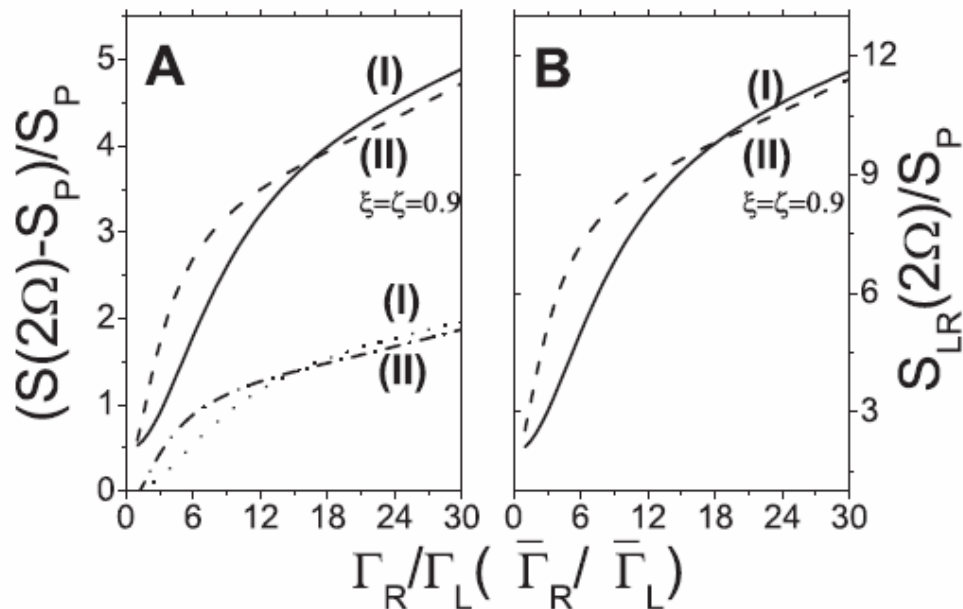
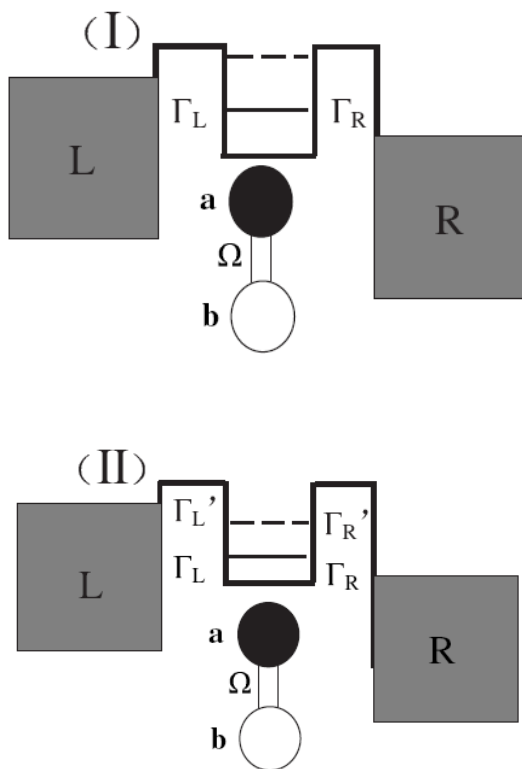
The two conditions needed to reach the “**Heisenberg efficiency**”:

$$(a) \operatorname{Re} S_{QI} = 0, \quad (b) |S_{QI}|^2 = S_Q S_I.$$

They indicate no lost information either through
(a) **phase** or (b) **energy** averaging

重新研究了 **single-dot SET** 测量量子比特这一重要问题，发现在**非线性响应区**，增强SET两个隧穿结的非对称性，可以提高测量结果的信噪比（SNR: Signal-to-Noise Ratio），并可以打破任意线性响应量子测量器件的 $\text{SNR} \leq 4$ 这一个由量子力学基本原理限定的极值结果。

H.J. Jiao *et al*, “Weak Measurement of Qubit Oscillations with Strong Response Detectors: Violation of the Fundamental Bound Imposed on Linear Detectors”, Phys. Rev. B 79, 075320 (2009)

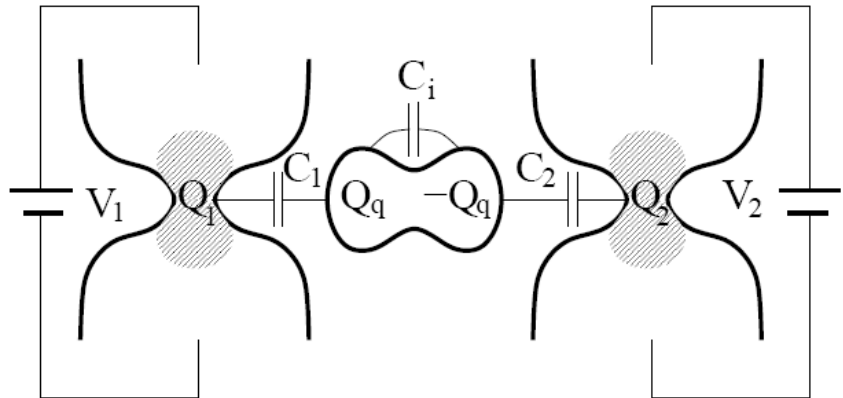
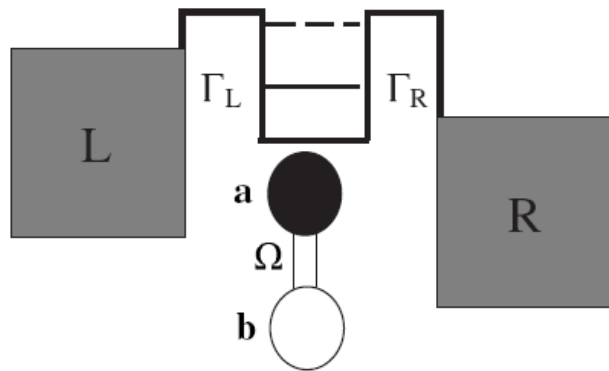


Understanding the violation of Korotkov-Averin bound

$$I(t) = aI_L(t) + bI_R(t)$$

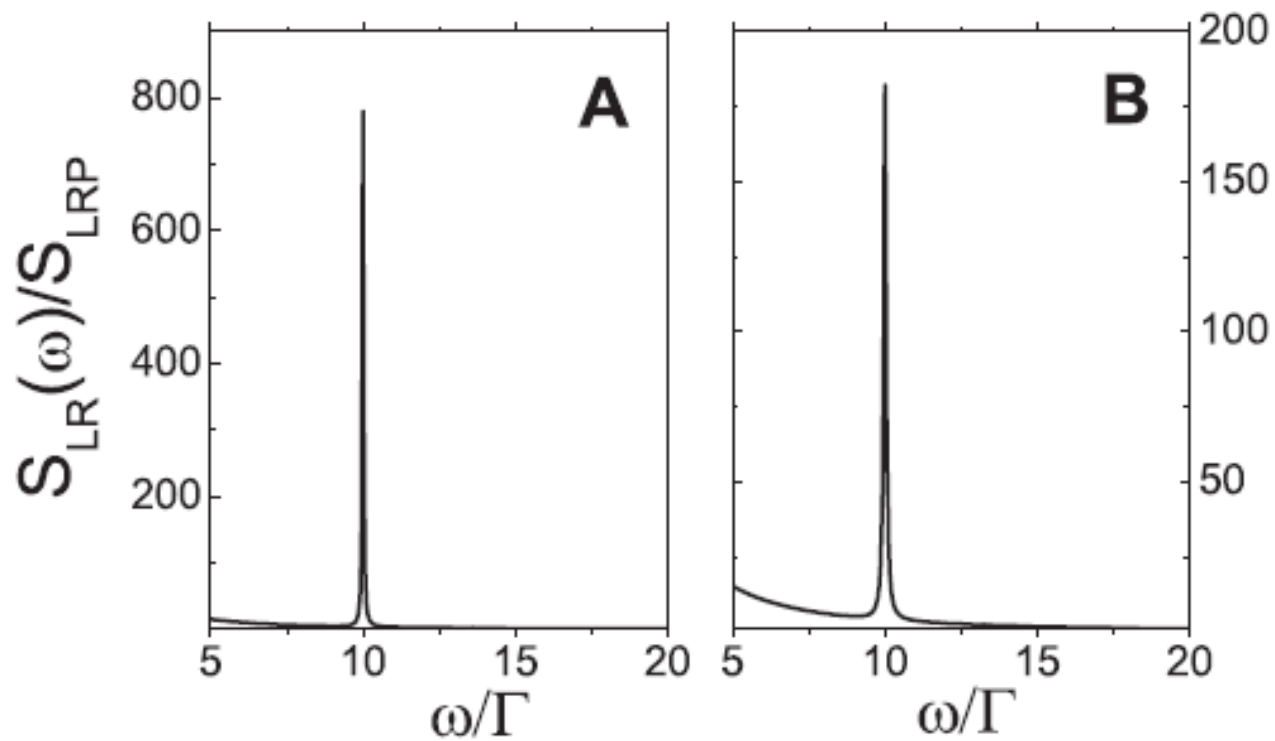
$$\langle I(t)I(0) \rangle: \quad S_{LR}(t) = \langle I_L(t)I_R(0) + I_R(t)I_L(0) \rangle$$

Cross correlation function



A. N. Jordan and M. Buttiker, Phys. Rev. Lett. 95, 220401 (2005).

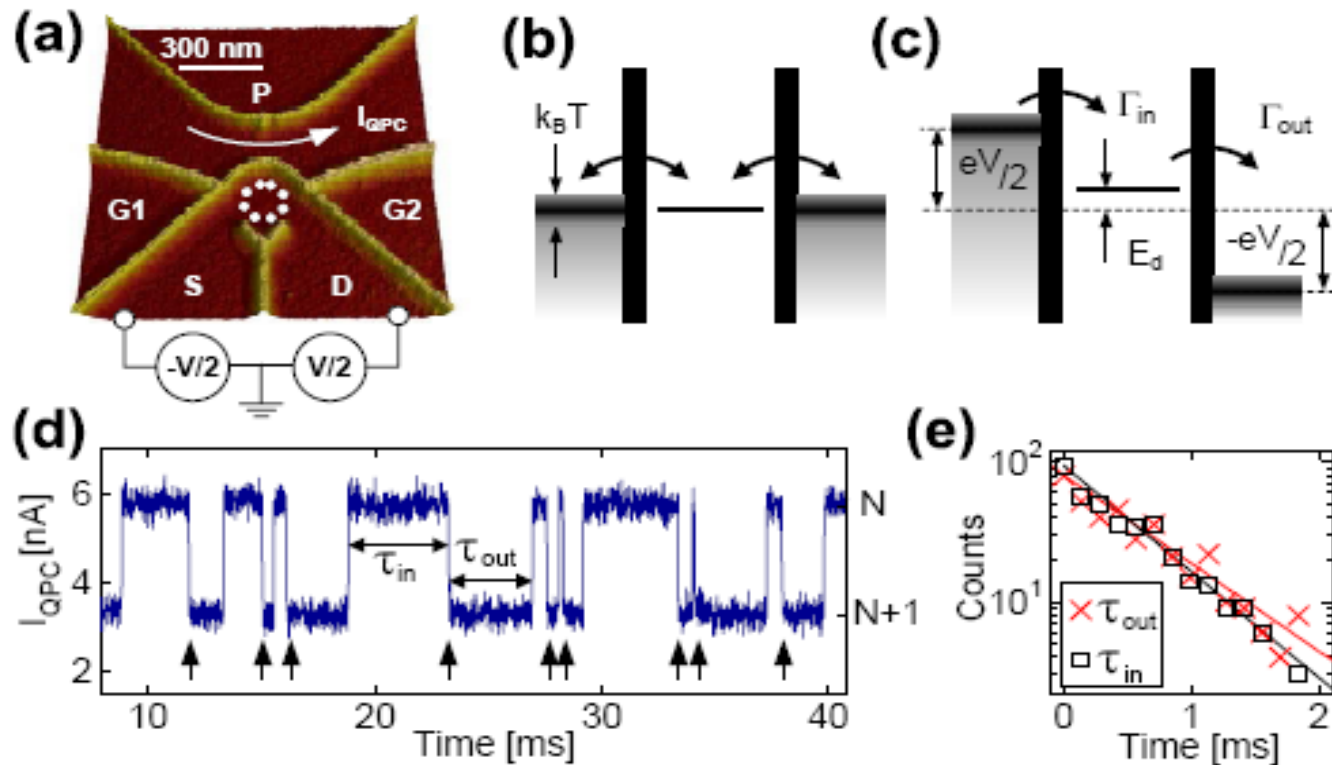
交叉关联噪声谱: 极大的“信噪比”, 可用于实际测量



量子输运方面的应用

K. Ensslin & A.C. Gossard *et al*, PRL 96, 076605 (2006)

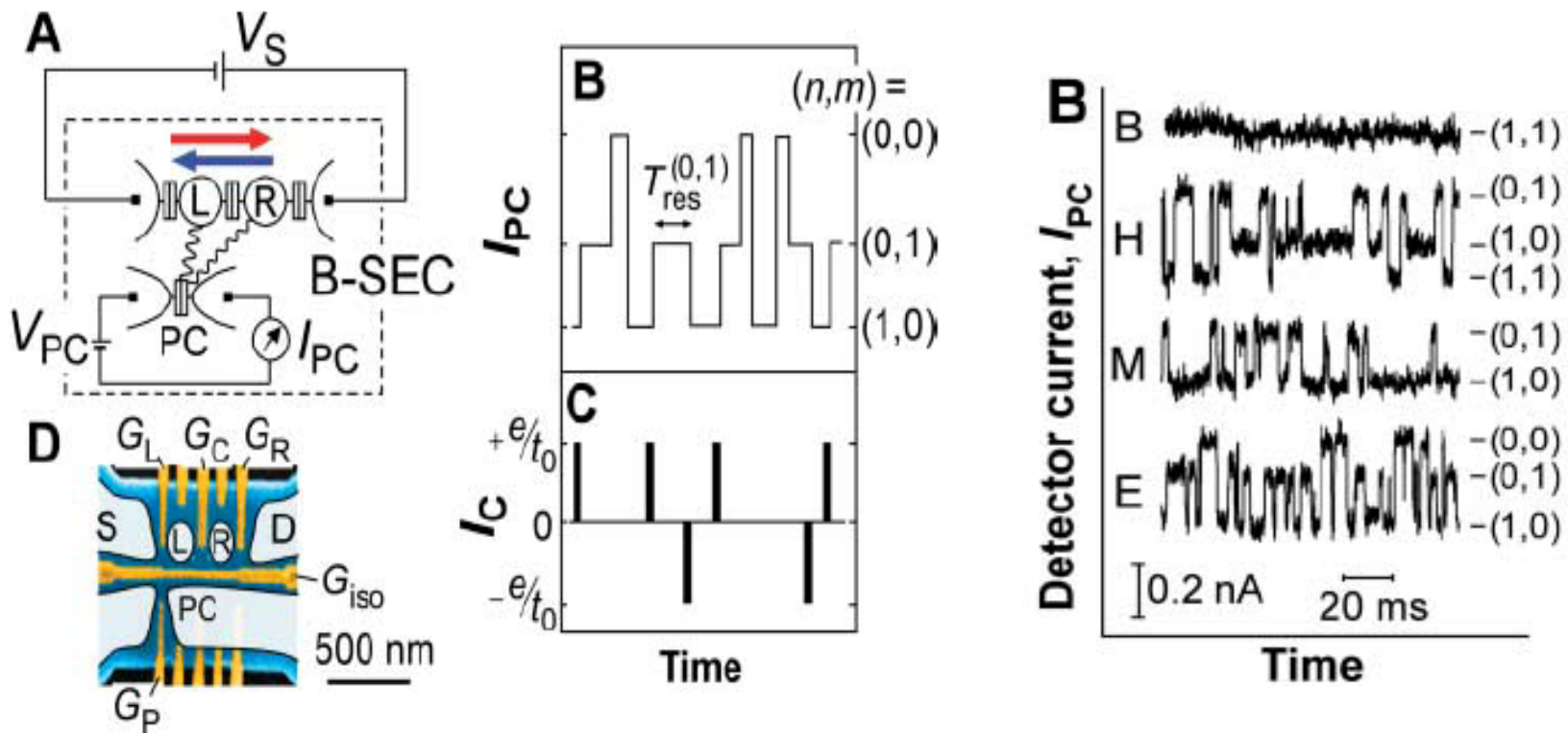
Counting Statistics of Single Electron Transport in a Quantum Dot



Science 310,
1634 (2006)

Bidirectional Counting of Single Electrons

Toshimasa Fujisawa,^{1,2*} Toshiaki Hayashi,¹ Ritsuya Tomita



Quantum mechanical complementarity probed in a closed-loop Aharonov–Bohm interferometer

DONG-IN CHANG¹, GYONG LUCK KHYM^{1,2}, KICHEON KANG², YUNCHUL CHUNG^{3*}, HU-JONG LEE^{1,4*}, MINKY SEO³, MOTY HEIBLUM⁵, DIANA MAHALU⁵ AND VLADIMIR UMANSKY⁵

¹Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Republic of Korea

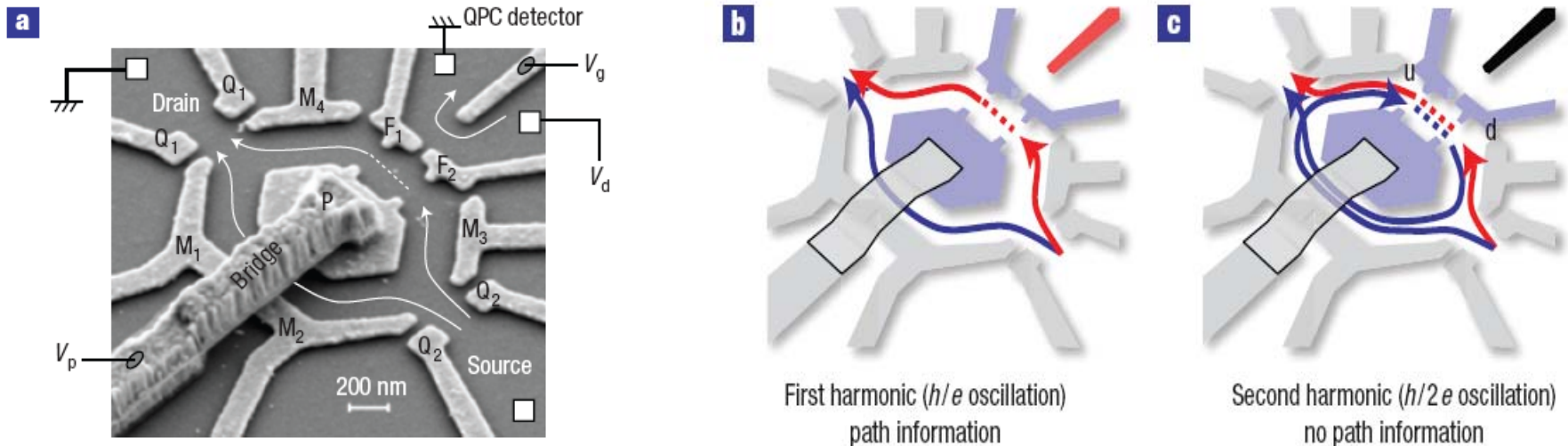
²Department of Physics, Chonnam National University, Gwangju 500-757, Republic of Korea

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Application to quantum transport:

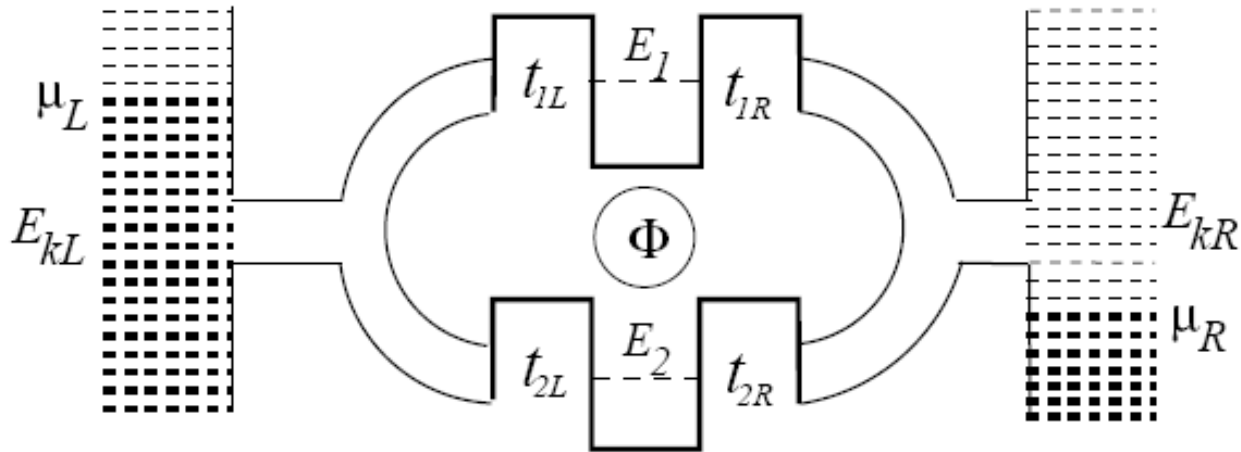
- 1) **Single and double QDs:** *Coulomb staircase, noise spectrum*
X.Q. Li et al, PRB 71, 205304 (2005)
J. Y. Luo et al, PRB 76, 085325 (2007)

- 2) **QD coupled to FM electrodes:** *spin-dependent current & fluctuations*
J. Y. Luo et al, J. Phys.:Condens.Matter 20, 345215 (2008)

- 3) **Transport through parallel quantum dots:** *counting statistics, magnetic field switching of current, giant fluctuations of current, harmonic decomposition of the interference pattern*
S.K. Wang et al, PRB 76, 125416 (2007)
F. Li, X.Q.Li, W.M. Zhang, and S.A. Gurvitz:
Europhys. Lett. 88, 37001(2009)
F. Li et al, Physica E 41, 521 (2009)
F. Li et al, Physica E 41, 1707(2009)

Quantum Transport through Parallel Quantum Dots

F. Li, X.Q.Li, W.M. Zhang, and S.A. Gurvitz: *Europhys. Lett.* **88**, 37001(2009)



$$H = H_0 + H_T + \sum_{\mu=1,2} E_{\mu} d_{\mu}^{\dagger} d_{\mu} + U d_1^{\dagger} d_1 d_2^{\dagger} d_2$$

$$H_0 = \sum_k [E_{kL} a_{kL}^{\dagger} a_{kL} + E_{kR} a_{kR}^{\dagger} a_{kR}]$$

$$H_T = \sum_{\mu,k} (t_{\mu L} d_{\mu}^{\dagger} a_{kL} + t_{\mu R} a_{kR}^{\dagger} d_{\mu}) + H.c.$$

$$t_{\mu L(R)} = \bar{t}_{\mu L(R)} e^{i\phi_{\mu L(R)}}$$

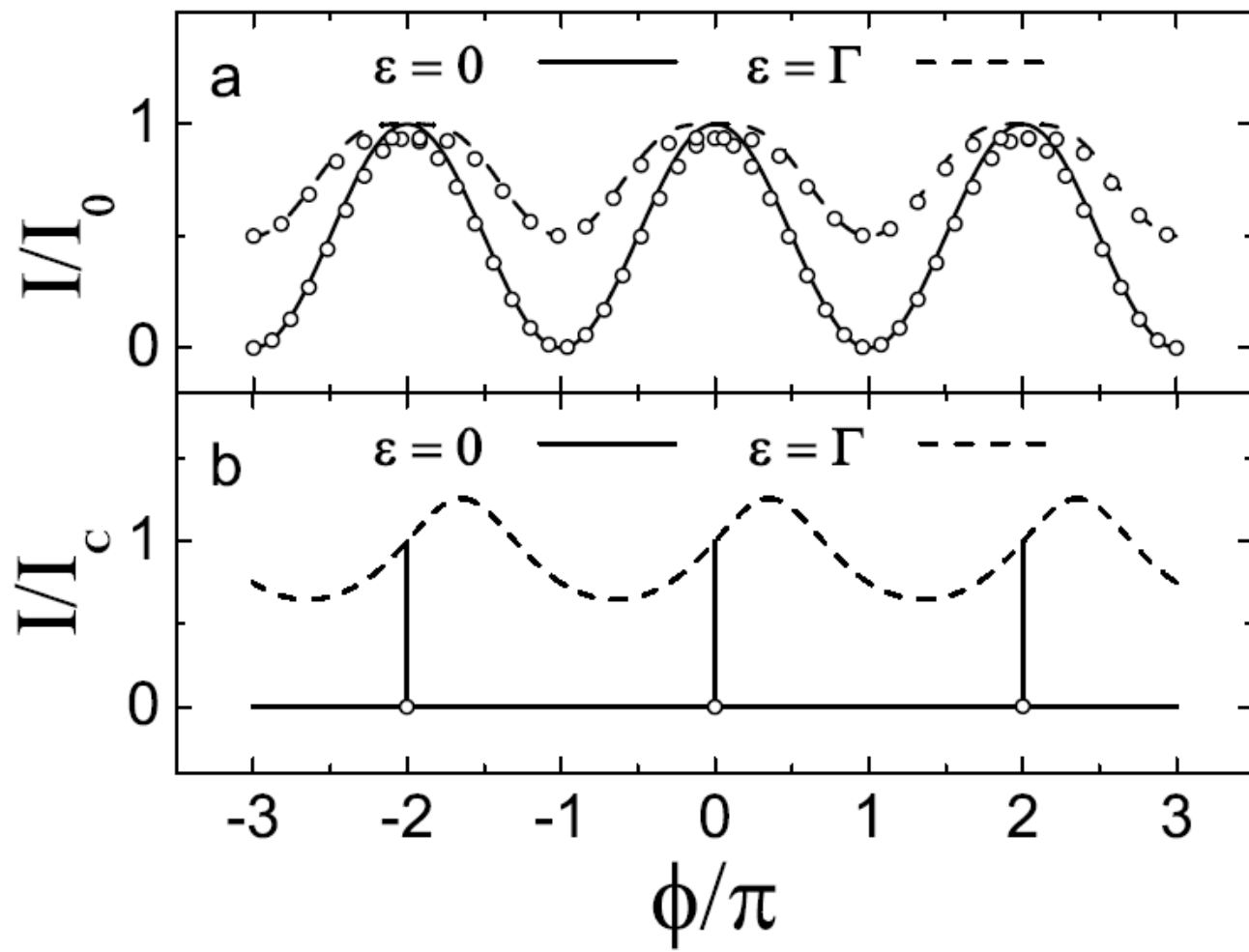
$$\phi_{1L} + \phi_{1R} - \phi_{2L} - \phi_{2R} = \phi$$

$$\phi \equiv 2\pi\Phi/\Phi_0$$

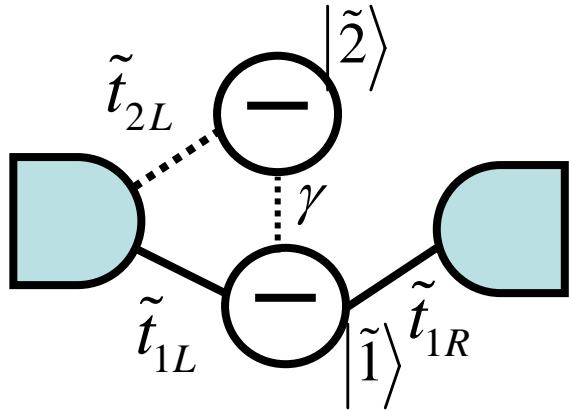
$$\begin{aligned}
\dot{\sigma}_{00} &= -2\Gamma_L\sigma_{00} + \Gamma_R(\sigma_{11} + \sigma_{22} + \bar{\sigma}_{12} + \bar{\sigma}_{21}) \\
\dot{\sigma}_{11} &= \Gamma_L\sigma_{00} - \Gamma_R\sigma_{11} - \Gamma_R(\bar{\sigma}_{12} + \bar{\sigma}_{21})/2 \\
\dot{\sigma}_{22} &= \Gamma_L\sigma_{00} - \Gamma_R\sigma_{22} - \Gamma_R(\bar{\sigma}_{12} + \bar{\sigma}_{21})/2 \\
\dot{\bar{\sigma}}_{12} &= e^{i\phi}\Gamma_L\sigma_{00} - \frac{\Gamma_R}{2}(\sigma_{11} + \sigma_{22}) - (i\epsilon + \Gamma_R)\bar{\sigma}_{12}
\end{aligned}$$

$$I(\phi) = I_C \frac{\epsilon^2}{\epsilon^2 + I_C \left(2\Gamma_R \sin^2 \frac{\phi}{2} - \epsilon \sin \phi \right)}$$

$$I_C = 2\Gamma_L\Gamma_R / (2\Gamma_L + \Gamma_R)$$



State basis transformation:



$$\gamma = \varepsilon \bar{t}_{1R} \bar{t}_{2R} / (\bar{t}_{1R}^2 + \bar{t}_{2R}^2)$$

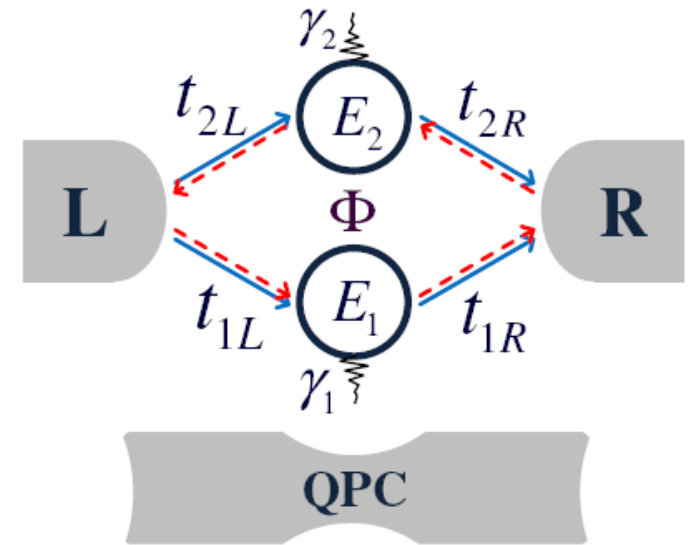
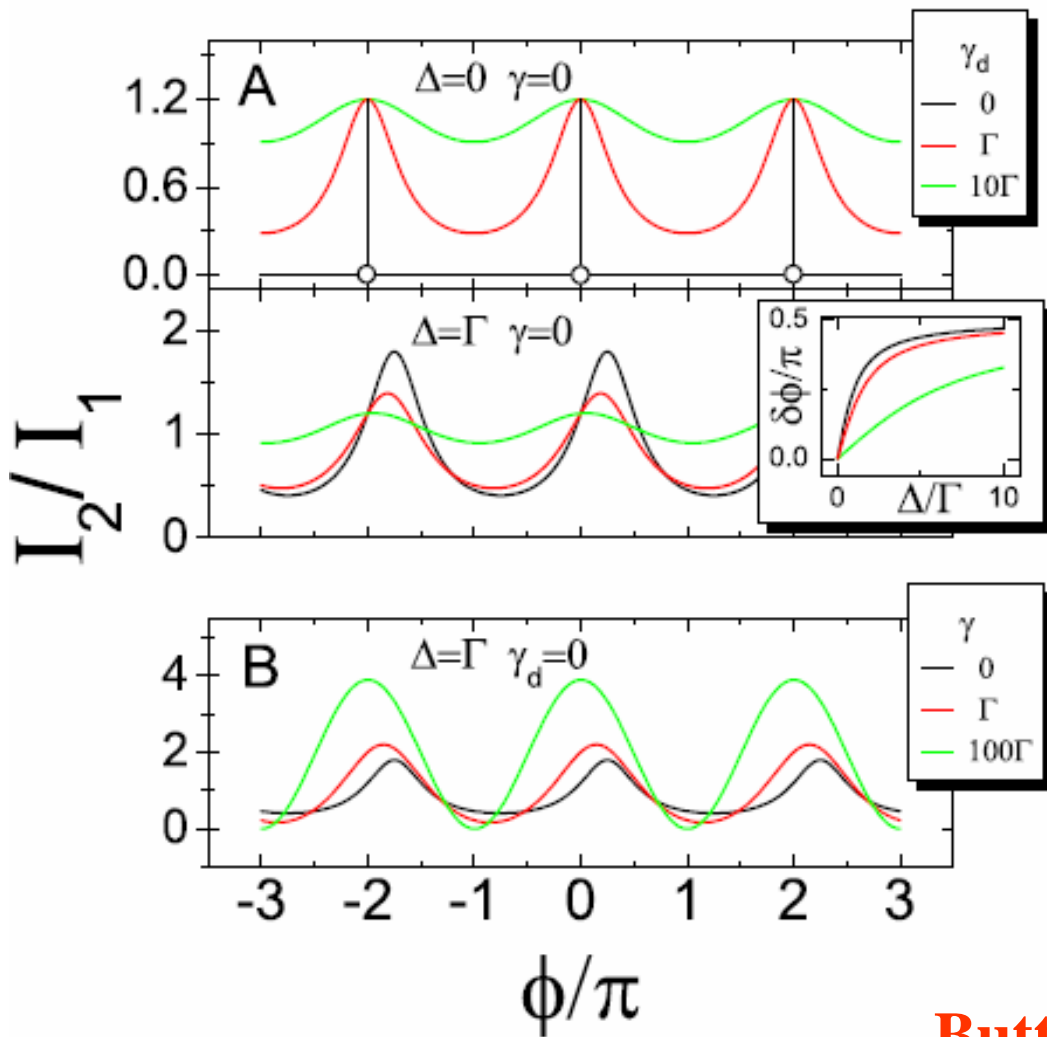
$$\begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix} = \frac{1}{\mathcal{N}} \begin{pmatrix} t_{1R} & t_{2R} \\ -t_{2R}^* & t_{1R}^* \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

$$\mathcal{N} = (\bar{t}_{1R}^2 + \bar{t}_{2R}^2)^{1/2}$$

(I) $\tilde{t}_{2R} = 0$

$$\tilde{t}_{2L}(\phi) = -e^{i(\phi_{2L} - \phi_{1R})} (\bar{t}_{1L} \bar{t}_{2R} e^{i\phi} - \bar{t}_{2L} \bar{t}_{1R}) / \mathcal{N}$$

(II) $\tilde{t}_{2L} = 0$ for $\phi = 2n\pi$
provided that $\bar{t}_{1L} / \bar{t}_{2L} = \bar{t}_{1R} / \bar{t}_{2R}$



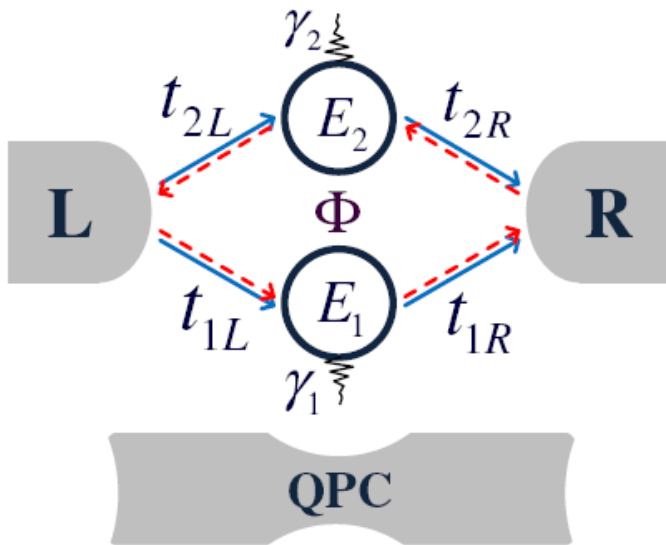
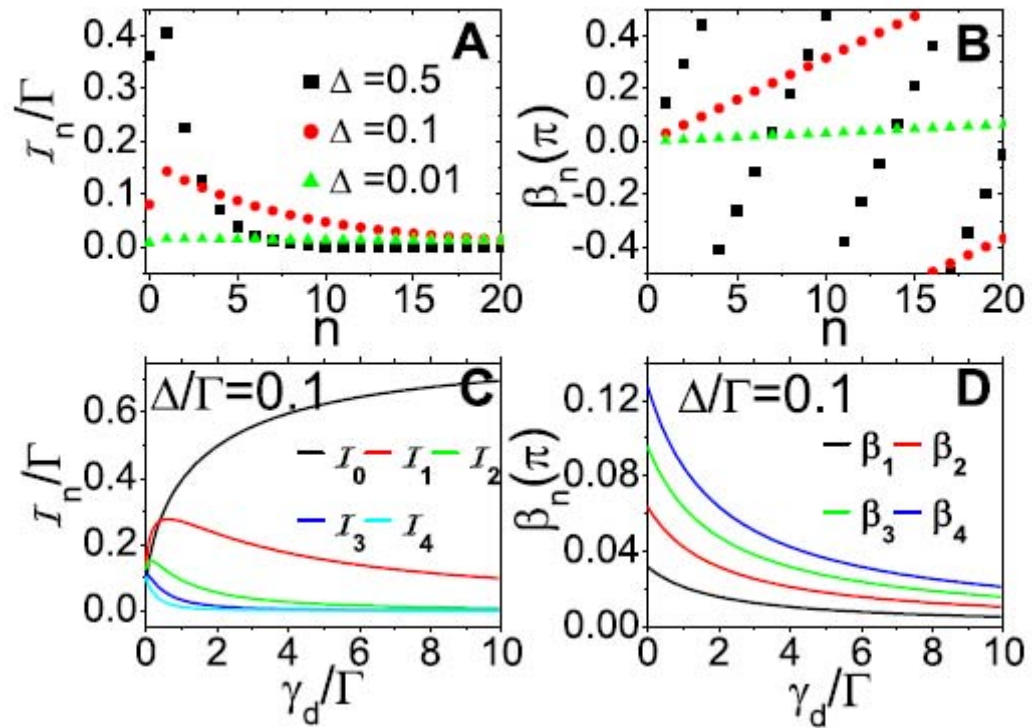
Visibility, dephasing
Close vs open ...
Magnetic asymmetry

Buttiker, PRL 57, 1761 (1986)

Based on **current conservation and time-reversal invariance**, the **Onsager relation**, say, the symmetry relation of transport coefficients under inversion of magnetic field, will lock the current peaks at ..., for *any two-terminal linear transport*.

Harmonic Analysis

$$I(\phi) = I_0 + \sum_{n=1}^{\infty} I_n \cos(n\phi + \beta_n)$$



(1) $\beta_n = n\beta_1$

(2) 1st partial wave $\propto t_2 e^{i\chi_2}$
 2nd partial wave $\propto t_1 e^{i\chi_1} t_2^* e^{i\chi_2} t_1 e^{i\tilde{\chi}_1}$

Quantum mechanical complementarity probed in a closed-loop Aharonov–Bohm interferometer

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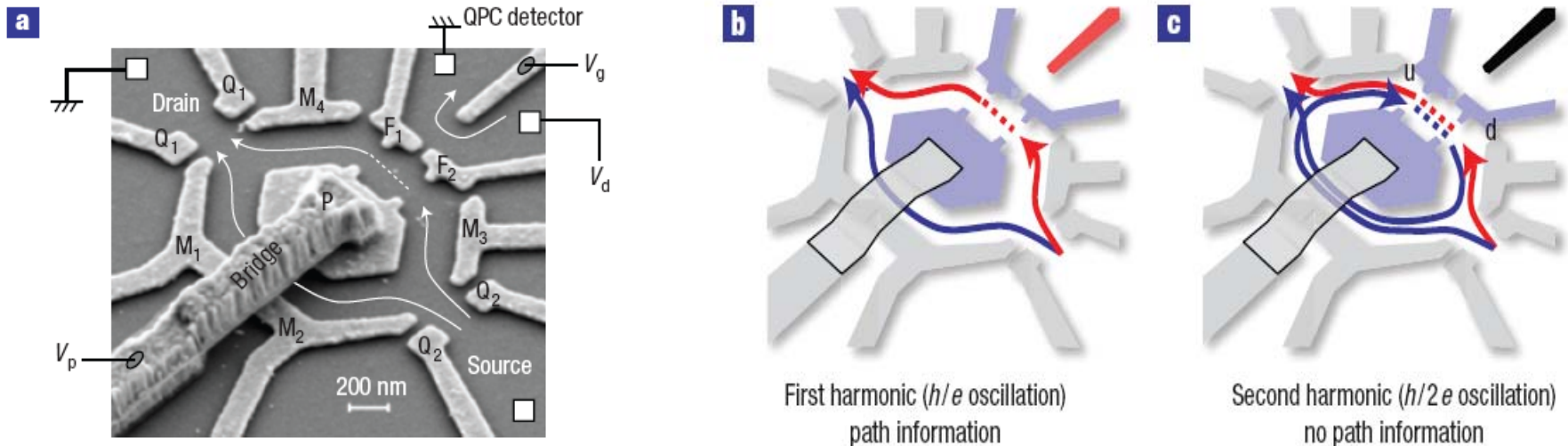
²Department of Physics, Chonnam National University, Gwangju 500-757, Republic of Korea

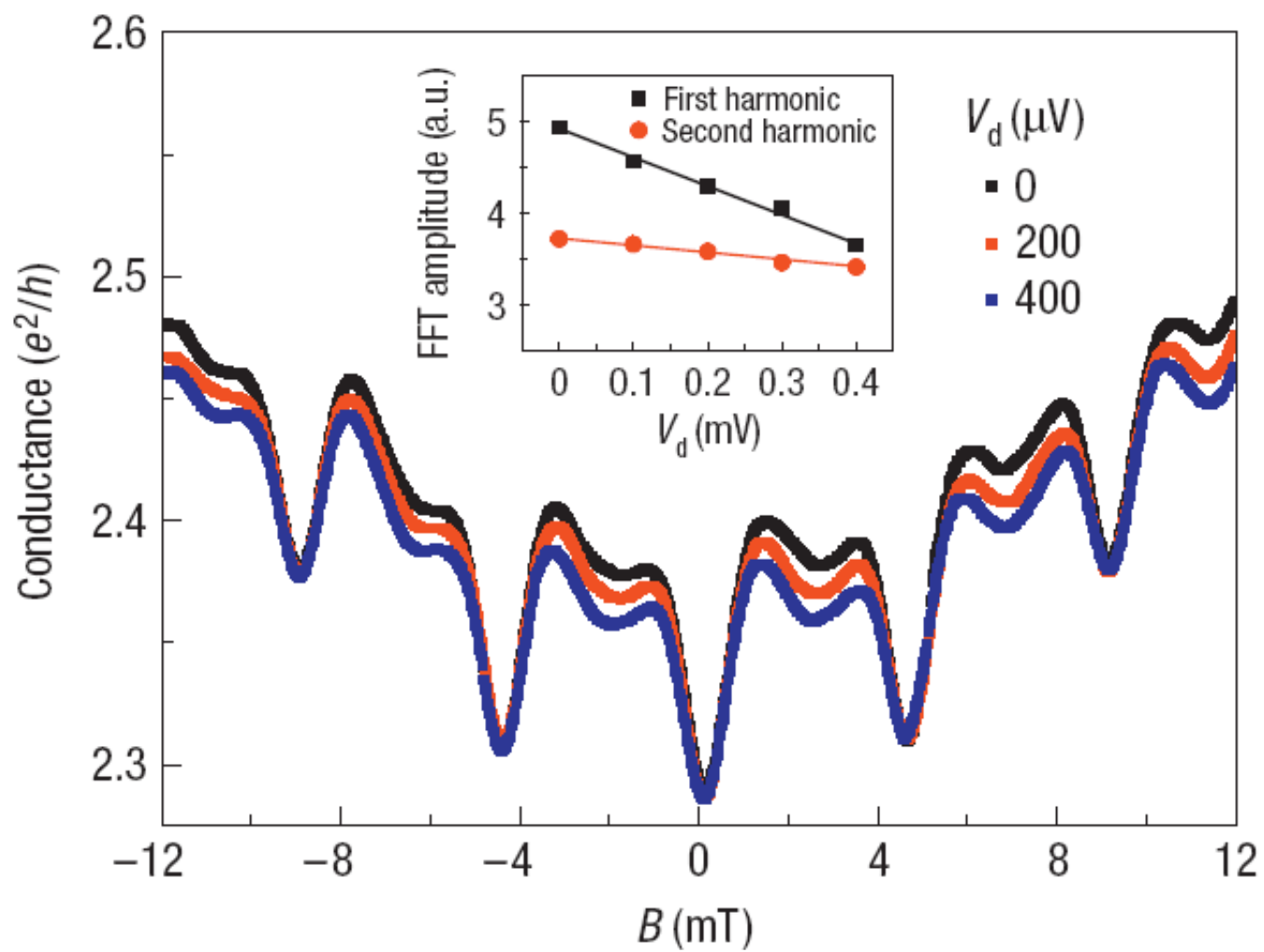
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Strongly super-Poissonian fluctuations, giant Fano factor

$$S_R(\omega) = \frac{8\Gamma_L\Gamma_R[2\Gamma_L\Gamma_R\Delta^2 - \Delta^4 + 3\Delta^2\omega^2 - 2\omega^2(\Gamma_R^2 + \omega^2)]\bar{I}}{[(2\Gamma_L + \Gamma_R)\Delta^2 - (2\Gamma_L + 3\Gamma_R)\omega^2]^2 + \omega^2(2\Gamma_L\Gamma_R + 2\Gamma_R^2 + \Delta^2 - \omega^2)^2} + 2\bar{I}$$

Limiting order:

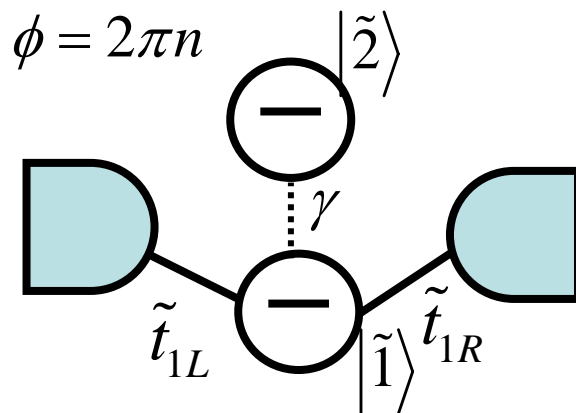
(i) First $\omega \rightarrow 0$ then $\Delta \rightarrow 0$

$$F \equiv \frac{S_R(0)}{2\bar{I}} = \frac{8\Gamma_L^2\Gamma_R^2 + (4\Gamma_L^2 + \Gamma_R^2)\Delta^2}{(2\Gamma_L + \Gamma_R)^2\Delta^2}$$

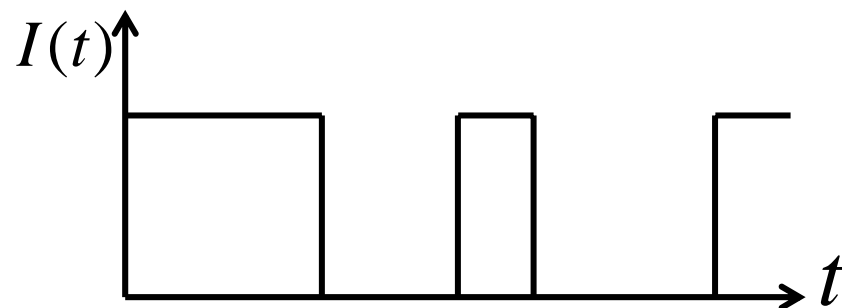
Divergent !!

(ii) First $\Delta \rightarrow 0$ then $\omega \rightarrow 0$

$$F = \frac{\Gamma_L^2 + \Gamma_R^2}{(\Gamma_L + \Gamma_R)^2}$$



$$\gamma \propto \Delta = |E_1 - E_2|$$



Enhanced Shot Noise in Tunneling through a Stack of Coupled Quantum Dots

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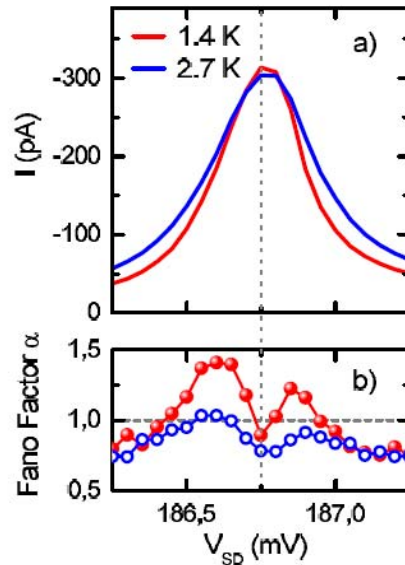
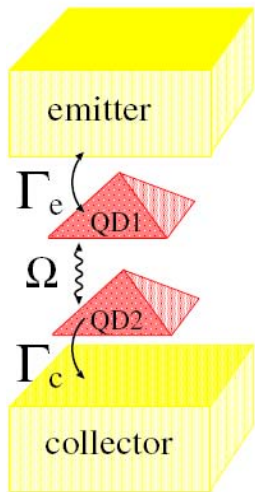
¹Institut für Festkörperphysik, Universität Hannover, Appelstrasse 2, D-30167 Hannover, Germany

²Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, United Kingdom

³Physikalisch-Technische Bundesanstalt, Bundesallee 100, D-38116 Braunschweig, Germany

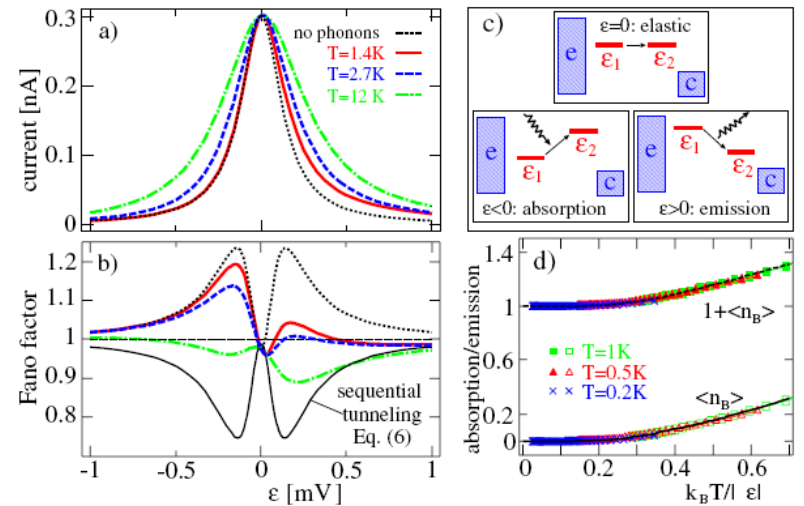
(Received 9 February 2006; published 21 June 2006)

We have investigated the noise properties of the tunneling current through vertically coupled self-assembled InAs quantum dots. We observe super-Poissonian shot noise at low temperatures. For increased temperature this effect is suppressed. The super-Poissonian noise is explained by capacitive coupling between different stacks of quantum dots.



PRL 99, 206602 (2007)

PHYSICAL REVIEW LETTERS



Summary

- ◆ **Master equation approach to quantum transport**
- ◆ **Quantum measurement of solid-state qubit**
 - **Example: SET detector, signal-to-noise ratio, etc**
- ◆ **Quantum transport**
 - **Current fluctuation, counting statistics**
 - **Example: double-dot interferometer**